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Bargaining with a Property Rights Owner

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Abstract

We consider a bargaining problem where one of the players, the *intellectual property rights owner* (IPRO) can allocate licenses for the use of this property among the interested parties (*agents*). The agents negotiate with him the allocation of licenses and the payments of the licensees to the IPRO. We state five axioms and characterize the bargaining solutions which satisfy these axioms. In a solution every agent obtains a weighted average of his individually rational level and his marginal contribution to the set of all players, where the weights are the same across all agents and all bargaining problems. The IPRO obtains the remaining surplus. The symmetric solution is the nucleolus of a naturally related coalitional game.

JEL classification: C78; C71; D45

Keywords: cooperative solution; nucleolus; patent licensing; intellectual property

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1 Introduction

Licensing is a common practice of disseminating an intellectual property among interested parties which allows an intellectual property rights owner (thereafter, *IPRO*) to receive revenue in the form of monetary transfers from the licensees. Since a license fee need not be uniform, i.e., the terms may be negotiated individually, a natural question arises: *Who should obtain the license and how to charge each licensee?* The value of a license for each interested party depends on who else obtains the license, thus the problem presents significant complexities.[1]

The paper deals with an owner of intellectual property rights (IPRO) and potential users of this property. A specific context is an innovator of a new technology which is superior to that used by firms in an oligopolistic industry. The IPRO can be either an incumbent firm or an independent research lab. He can sell licenses for the use of his new technology to any subset of firms. Every allocation of licenses determines the payoffs of the IPRO and the firms in the industry. We provide a normative (axiomatic) approach to the bargaining between the IPRO and the firms in the industry about the allocation of licenses and monetary transfers of the firms in return.

A bargaining solution is a mapping which associates with every bargaining problem a vector of net payoffs to all players. Indirectly, a solution determines the allocation of licenses and their transfers to the IPRO. We study solutions which satisfy certain requirements (axioms). Our first axiom asserts that a solution should be undominated. Namely, for every subset of firms, there is no other outcome that makes the IPRO and every member of this subset strictly better off. The second axiom requires that if two bargaining problems have the same sets of undominated outcomes, then they must have the same solution.

[1] For instance, a Vickrey auction need not be efficient because of presence of the externalities in bidders' values.

This axiom is similar in spirit to the well known axiom of independence of irrelevant alternatives (Nash, 1950). It asserts that the dominated outcomes are "irrelevant" and thus should not affect the solution.[2] The third axiom states that a solution should not depend on the unit of measurement. The fourth axiom requires that a solution should not depend on the names of the agents. The last axiom deals with bargaining problems that are composed of two independent industries with two different sets of firms. The axiom requires that in this case the net payoff of a firm should depend only on its industry.

We show that in every solution which satisfies the above five axioms the IPRO allocates licenses efficiently (that is, the license allocation maximizes the total industry profit) and every firm's net payoff is a *weighted* average of its individually rational level, the amount that it can guarantee irrespective of a licence allocation, and its marginal contribution to the grand coalition. The IPRO obtains the remaining surplus. Furthermore, these weights are the same across all firms and across all bargaining problems with any finite number of firms. The weights therefore serve as a measure of the bargaining power of the IPRO. They are completely determined by the simple one-firm problem, where the firm receives zero without the license and one with it, and the IPRO, who is an outside lab, can obtain by himself only zero. This can be regarded as a symmetric problem: The IPRO and the firm can each achieve zero by themselves and could obtain one together. If the solution of this specific problem is that the IPRO and the firm obtain α and $1-\alpha$, respectively, then the solution of *every* bargaining problem with any number of firms awards every firm the average of its individually rational level and its marginal contribution to the grand coalition with the same weights $(\alpha, 1-\alpha)$. A special case, the symmetric solution with $\alpha = 1/2$, coincides with the *nucleolus* (Schmeidler,

[2] A conceptual difference between our axiom and the standard IIA axiom is that in the latter the notion of "irrelevant outcome" depends on a given solution (see the discussion in the text, Section 5).

1969) of a naturally related coalitional game.

Though we focus on patent licensing, this paper can be applied to more general bargaining problems, where one "powerful" player (a monopolist or a bureaucrat) has the power to dictate any outcome in a given set of feasible outcomes. One example is an n-player bargaining over a split of a cake where an additional player, an arbitrator, has the exclusive power to dictate any allocation. Another example deals with an information holder who exclusively owns a piece of information relevant to the players in a strategic conflict. He has many ways to transmit part of his information (or all of it) to some (or all) players (see, e.g., Kamien, Tauman, and Zamir, 1990). The information holder may bargain with the players about the information to be transmitted to each agent and about their monetary transfers. Another application concerns a group of lobbyists (with, potentially, conflicting interests) offering rewards to a policy maker if their desired policy is implemented.

Our framework resembles that of Buch and Tauman (1992) who deal with similar bargaining problems. Their work, however, is confined to the special case where the powerful player has no stake in the bargaining, and his only source of income is the agents' transfers. These problems do not apply, for instance, to patent licensing problems where the patent holder is an incumbent firm. Our axiomatic approach is different from that of Buch and Tauman, and we argue that our solution is more appealing.

Throughout the paper we assume that the set of outcomes is commonly known. Bernheim and Whinston (1986) (thereafter, BW) consider a similar framework with asymmetric information, where the powerful player (the auctioneer, in BW) has no information about the agents' preferences.[3] The bargaining problem is resolved by an auction. Every agent submits a contingent schedule which specifies the transfer of the agent to the auctioneer as a

[3]Even though the agents themselves are fully informed. BW note that relaxation of this assumption leads to significant complexities.

function of the dictated outcome. The schedules are selected simultaneously and they are assumed to be commitments. After observing these schedules, the auctioneer dictates an outcome and collects the corresponding transfers. The BW paper focuses on truthful[4] Nash equilibrium points. It can be shown that in every (submodular) bargaining problem the unique truthful Nash equilibrium outcome coincides with our extreme solution, where the bargaining power of the powerful player is minimal.

As for the application to patent licensing, a plethora of works approach this problem noncooperatively, employing as pricing mechanisms upfront fees, royalties, auctions, and their combinations (see Kamien, 1992, for a comprehensive survey of early literature; see also Sen and Tauman, 2007, and the references within). Tauman and Watanabe (2007), perhaps, is the only exception which uses instead a normative approach, where the licensing process is considered as a bargaining problem between the IPRO and the firms, with semi-transferrable utilities (only transfers from the firms to the IPRO are allowed). Tauman and Watanabe consider the Shapley value as a bargaining solution and show that asymptotically it coincides with the non-cooperative results.

2 Notations and Definitions

Our model deals with an infinite set of potential agents and an *intellectual property rights owner* (*IPRO*). We denote by $\mathbb{Z} = \{1, 2, \ldots\}$ the set of agents and by 0 the IPRO. A bargaining problem is a pair (N^0, X), where $N^0 = N \cup \{0\}$, N is a finite subset of $\mathbb{Z}$, and X is a nonempty compact subset of

[4] A truthful strategy of an agent in BW is a contingent plan which is characterized by a real number y. The transfer to the monopolist is the difference between the gross payoff of the agent and y, as long as this difference is positive; otherwise, the transfer is zero. A truthful Nash equilibrium is a Nash equilibrium where every agent plays a truthful strategy.

$\mathbb{R}_+^{N^0}$ (finite or infinite) of all possible bargaining outcomes. Every outcome x in X is a *gross payoff vector* for the players in N^0. The IPRO (and only the IPRO) has the ability to dictate any outcome in X. The agents in N bargain with the IPRO about the outcome to be dictated and, as a result, transfer to the IPRO some parts of their gross payoffs. Thus, the bargaining is on both: the outcome in X and the transfers of the agents. It is assumed that only agreements with the IPRO are enforceable. Agents may or may not be allowed to transfer payoffs from one to another. If such transfers are allowed, then the projection of X on N is a simplex.

Let (N^0, X) be an $(n+1)$-player bargaining problem, that is, $|N^0| = n+1$. For simplicity, we will always assume that $N^0 = \{0, 1, \ldots, n\}$. Denote by $\mathcal{X}_{n+1}$ the class of all $(n+1)$-player bargaining problems, and let $\mathcal{X} = \bigcup_{k=1}^{\infty} \mathcal{X}_k$.

For $(N^0, X) \in \mathcal{X}$, suppose that an outcome $x \in X$, $x = (x_0, x_1, \ldots, x_n)$, is dictated. Then every agent $i \in N$ obtains the gross payoff x_i and pays z_i, $0 \leq z_i \leq x_i$, to the IPRO, thus receiving the *net payoff* $y_i = x_i - z_i$. The IPRO receives the *net payoff* $y_0 = x_0 + \sum_{i \in N} z_i$. Let $y = (y_0, y_1, \ldots, y_n)$.

It is important to note that the IPRO must select an outcome in X no matter whether he reaches an agreement with the agents or not. If the IPRO has an option to do nothing, then the "inaction" outcome must be in X.

For any subset $S \subset N$ let $S^0 = S \cup \{0\}$. An outcome $x^* \in X$ is said to be *efficient for* $S^0 \subset N^0$ if

$$\sum_{i \in S^0} x_i^* = \max_{x \in X} \sum_{i \in S^0} x_i.$$

It is called *efficient* if it is efficient for N^0. For every $S^0 \subset N^0$ denote

$$E_{S^0}(X) = \{x \in X \mid x \text{ is efficient for } S^0\}$$

and let $E(X) = E_{N^0}(X)$.

For a bargaining problem (N^0, X), the *individually rational level* $d_i(X)$ of an agent $i \in N$ is the gross payoff that i can guarantee to obtain. Formally, the individually rational level of the IPRO is

$$d_0(X) = \max\{x_0 : x \in X\},$$

The individually rational level of every agent i is the gross payoff guaranteed to the agent irrespective of the dictated outcome[5]

$$d_i(X) = \min\{x_i : x \in X\}, \quad i \in N.$$

Definition Let $(N^0, X) \in \mathcal{X}$. A net payoff vector $y = (y_0, y_1, \ldots, y_n)$ is *feasible for* $S^0 \subset N^0$ *at* $x \in X$ if

(i) $y_i \geq d_i(X)$ for every $i \in S^0$,

(ii) $y_i \leq x_i$ for every $i \in S$ and $y_j = x_j$ for every $j \in N \backslash S$,

(iii) $\sum_{i \in S^0} y_i = \sum_{i \in S^0} x_i$.

A net payoff vector y is *feasible for* S^0 if it is feasible for S^0 at some $x \in X$. A net payoff vector y is *feasible* if it is feasible for N^0.

Condition (i) requires that every player in S^0 obtains at least his individually rational level; (ii) requires that only transfers *from the agents in* S *to the IPRO* are allowed (and agents not in S obtain their gross payoffs); condition (iii) requires that the total payoff of S^0 obtained from an outcome x is distributed entirely among the players in S^0, i.e., nothing is transferred to an outside party or wasted.

Let $(N^0, X) \in \mathcal{X}$ and $x \in X$. Denote by $Y(x)$ the set of net payoff vectors which are feasible at x and let $Y(X)$ be the set of net payoff vectors which are feasible for X, i.e., $Y(X) = \bigcup_{x \in X} Y(x)$.

[5] Alternative definitions of the individual rationality that do not change the resuts of the paper are discussed in Remark 2 (Section 6) below.

3 Stability

Let (N^0, X) be a bargaining problem in $\mathcal{X}$. Let $S \subset N$, $S^0 = S \cup \{0\}$, and $y, y' \in Y(X)$. We say that y' *dominates* y *via* S^0 if y' is feasible for S^0 and $y'_i > y_i$ for all $i \in S^0$.

Definition A payoff vector $y \in Y(X)$ is *stable* if it is undominated, that is, if for every $S^0 \subset N^0$ there is no $y' \in Y(X)$ which dominates y via S^0.

In other words, a payoff vector y is stable if the IPRO cannot find a subset S of agents and a feasible payoff vector y' for S^0 so that he and everyone in S are strictly better off.

Proposition 1 *Let $(N^0, X) \in \mathcal{X}$. A payoff vector $y \in Y(X)$ is stable if and only if for every $S^0 \subset N^0$*

$$\sum\nolimits_{i \in S^0} y_i \geq \max_{x \in X} \sum\nolimits_{i \in S^0} x_i.$$

Proof. Let $y \in Y(X)$ be non-stable, that is, there is $S \subset N$ and y' feasible for S^0 such that $y_i < y'_i$ for all $i \in S^0$. Hence, there is $x \in X$ such that

$$\sum\nolimits_{i \in S^0} y_i < \sum\nolimits_{i \in S^0} y'_i = \sum\nolimits_{i \in S^0} x_i.$$

Conversely, let $y \in Y(X)$ be stable. Suppose to the contrary that

$$\sum\nolimits_{i \in S^0} y_i < \sum\nolimits_{i \in S^0} \hat{x}_i$$

for some $S^0 \subset N^0$ and some $\hat{x} \in E_{S^0}(X)$. Let $T = \{j \in S^0 \mid y_j < \hat{x}_j\}$. Clearly, $T \neq \varnothing$ and $0 \notin T$ (if $y_0 < \hat{x}_0$, then y is dominated via $\{0\}$ by $y' \in \operatorname*{argmax}_{x \in X} x_0$). Define $w \in \mathbb{R}_+^{N^0}$ by

$$w_j = \begin{cases} y_j + \varepsilon, & j \in T, \\ \hat{x}_j, & j \in N \backslash T, \\ \hat{x}_0 + \sum_{j \in T}(\hat{x}_j - y_j - \varepsilon), & j = 0, \end{cases}$$

where $\varepsilon > 0$ is small enough, such that $w_j = y_j + \varepsilon < \hat{x}_j$ for all $j \in T$ and

$$y_0 + \sum_{j \in T} (y_j + \varepsilon) < \hat{x}_0 + \sum_{j \in T} \hat{x}_j \tag{1}$$

Since $d_j(X) \leq y_j < w_j < \hat{x}_j$ for all $j \in T$ and $\sum_{j \in T^0} w_j = \sum_{j \in T^0} \hat{x}_j$, w is feasible for T^0 at $\hat{x}$. But $w_j > y_j$ for all $j \in T$, and by (1)

$$w_0 = \hat{x}_0 + \sum_{j \in T} (\hat{x}_j - y_j - \varepsilon) > y_0.$$

Hence, y is dominated by w via T^0, a contradiction. ■

Denote by $ST(X)$ the set of stable net payoff vectors in a bargaining problem (N^0, X).

A payoff vector $y \in Y(X)$ is *efficient* if it is feasible at some efficient outcome in X, i.e., if there is $x^* \in E(X)$ such that $y \in Y(x^*)$.

Corollary 1 *If $y \in Y(X)$ is stable, then it is efficient.*

4 Related Games in Coalitional Form

A game (N^0, V) in coalitional form consists of the set N^0 of players and a function $V : 2^{N^0} \to \mathbb{R}$ such that $V(\varnothing) = 0$. Every $S \subset N^0$ is called a coalition and N^0 is called the grand coalition.

Let (N^0, X) be a bargaining problem in $\mathcal{X}$. We associate with (N^0, X) the game in coalitional form (N^0, V_X), for which the worth of every coalition S is the highest total payoff that it can *guarantee* to its members,

$$V_X(S) = \begin{cases} \max\limits_{x \in X} \sum\limits_{i \in S} x_i, & S \ni 0, \\ \sum\limits_{i \in S} d_i(X), & S \not\ni 0. \end{cases} \tag{2}$$

The *core* of (N^0, V_X) is denoted by $\mathcal{C}_{V_X}$ and is defined to be the set of all $y \in \mathbb{R}^{N^0}$ such that $\sum_{i \in S} y_i \geq V_X(S)$ for all $S \subset N^0$ and $\sum_{i \in N^0} y_i = V(N^0)$.

The following proposition shows that for every bargaining problem (N^0, X) in $\mathcal{X}$, the set of stable net payoff vectors $ST(X)$ coincides with the core of (N^0, V_X).

Proposition 2 *For every* $(N^0, X) \in \mathcal{X}$*,* $ST(X) = \mathcal{C}_{V_X}$*.*

The proof appears in the Appendix.

With slight abuse of notations, we shall often refer to the set $ST(X)$ as simply the core of bargaining problem (N^0, X).

For every $i \in N$ and every $S^0 \ni i$, denote by $MC_i(S^0, X)$ the *marginal contribution of* i *to the coalition* S^0,

$$MC_i(S^0, X) = V_X(S^0) - V_X(S^0 \backslash \{i\}).$$

A bargaining problem is called submodular if the marginal contribution of every agent to a coalition decreases with the coalition size (with respect to inclusion). Formally:

Definition A bargaining problem $(N^0, X) \in \mathcal{X}$ is *submodular* if for all $i \in N$ and all $S \supset T \ni i$

$$MC_i(S^0, X) \leq MC_i(T^0, X). \tag{3}$$

Denote by $\mathcal{X}^{SM}$ the class of submodular bargaining problems. Submodularity is the standard diminishing returns assumption. This class includes the problems with "cut-throat" competition, where the outcomes which benefit only one of the agents (and yield zero to the rest) are efficient. It is, for instance, n-player bargaining over a split of a cake where the $(n+1)$-st player, the IPRO, has the exclusive power to dictate allocation. Another example of a submodular bargaining problem is an interaction of a patent holder of a new technology and the firms in an oligopolistic industry. The patent holder can sell licenses to use his technology to any number of firms via up-front fees, royalties, or combinations of the two. An additional licensee firm increases the total industry profit, but in a decreasing rate. The larger is the number of licenses sold, the smaller is the marginal value of an additional license.

The following proposition asserts that every submodular bargaining problem has a nonempty core.

Proposition 3 *For every* $(N^0, X) \in \mathcal{X}^{SM}$, $ST(X)$ *is nonempty.*

We make use of the following lemma.

Lemma 1 *Let* $(N^0, X) \in \mathcal{X}^{SM}$. *Then* $y \in ST(X)$ *if and only if*

(i) $d_i(X) \leq y_i \leq MC_i(N^0, X)$ *for all* $i \in N$,

(ii) $y_0 = V_X(N^0) - \sum_{i \in N} y_i$.

Proof. Suppose that $y \in ST(X)$. Then (i) and (ii) are immediate by Proposition 2. Conversely, suppose that y satisfies (i) and (ii). By Proposition 2, to prove that $y \in ST(X)$ it suffices to show that for every $S \subset N$ $\sum_{i \in S^0} y_i \geq V_X(S^0)$. By (i) and (ii),

$$y_0 + \sum_{i \in S} y_i = V_X(N^0) - \sum_{j \in N \backslash S} y_j \geq V_X(N^0) - \sum_{j \in N \backslash S} MC_j(N^0, X),$$

and since $X \in \mathcal{X}^{SM}$ we have

$$\begin{aligned} \sum_{j \in N \backslash S} MC_j(N^0, X) &\leq MC_{j_1}(N^0, X) + MC_{j_2}(N^0 \backslash \{j_1\}, X) \\ &\quad + \ldots + MC_{j_{n-s}}(N^0 \backslash \{j_1, \ldots, j_{n-s-1}\}, X) \\ &= V_X(N^0) - V_X(S^0), \end{aligned}$$

where $\{j_1, j_2, \ldots, j_{n-s}\} = N \backslash S$. ■

Proof of Proposition 3. Consider point $y \in \mathbb{R}^{N^0}$ defined as follows:

$$y_j = \begin{cases} d_j(X), & j \in N, \\ V_X(N^0) - \sum\limits_{i \in N} d_i(X), & j = 0. \end{cases}$$

By Lemma 1, y is in $ST(X)$. ■

5 An Axiomatic Approach

In this section we define a solution on $\mathcal{X}^{SM}$ and present five axioms for a solution to satisfy.

Definition A *solution on* $\mathcal{X}^{SM}$ is a mapping, ϕ, which associates with every bargaining problem (N^0, X) in $\mathcal{X}^{SM}$ a payoff vector $\phi(X)$ in $Y(X)$.

We impose the following five axioms on ϕ. The first axiom requires that a solution of every problem is stable.

Axiom 1 (Stability) *For every* $(N^0, X) \in \mathcal{X}^{SM}$*,* $\phi(X) \in ST(X)$*.*

This assumes that the IPRO will reject a payoff vector y if he can reach another settlement y' with some subset of agents $S \subset N$ such that every member of S^0 is strictly better off with y' than with y. Note that by Corollary 1, if ϕ satisfies Axiom 1, then $\phi(X)$ is an efficient payoff vector.

The second axiom asserts that only stable net payoff vectors are relevant for the solution. That is, any net payoff vector which is not stable is not considered to be a credible settlement for the IPRO, thus it should not affect the solution.

Axiom 2 (Stability Dependence (STD)) *For every* (N^0, X) *and* (N^0, X') *in* $\mathcal{X}^{SM}$*, if* $ST(X) = ST(X')$*, then* $\phi(X) = \phi(X')$*.*

This axiom resembles the principle of *independence of irrelevant alternatives* (IIA). Any non-stable net payoff vector is "irrelevant", since the IPRO who has the power to dictate any outcome will reject those that can be improved upon. Thus the solution should not depend on "irrelevant" net payoff vectors. Note, however, that this axiom is not exactly analogous to Nash (1950)'s IIA. In the Nash bargaining problem, "irrelevance" of outcomes depends on both the specific problem and the given solution. Every outcome which is not the solution outcome is irrelevant in the sense that it could be deleted from the set of outcomes without affecting the solution.[6] In contrast, in our context an irrelevant outcome is determined only by the bargaining problem and not by the solution. Given a problem, the irrelevant outcomes are exactly those which are not stable, hence deleting or adding a non-stable outcome does not affect the solution.

Next, we require that a solution does not depend on the unit of measurement.

Axiom 3 (Scale Covariance) *For every* $(N^0, X) \in \mathcal{X}^{SM}$*, every* $b \in \mathbb{R}^{N^0}$ *and every scalar* $c > 0$*, if* $(N^0, cX + b) \in \mathcal{X}^{SM}$*, then*

$$\phi(cX + b) = c\phi(X) + b.$$

The next axiom requires that a solution does not depend on the names of the agents. Let $(N^0, X) \in \mathcal{X}^{SM}$ and let π be a permutation of $N = \{1, \ldots, n\}$. For every $x \in \mathbb{R}^n$, let $\pi x \in \mathbb{R}^n$ be such that $(\pi x)_i = x_{\pi(i)}$ for all $i \in N$ and let $\pi X = \{\pi x \mid x \in X\}$.

Axiom 4 (Anonymity) *Suppose that* $(N^0, X) \in \mathcal{X}^{SM}$*. For every permutation* π *of* N*, if* $(N^0, \pi X) \in \mathcal{X}^{SM}$*, then*

$$\phi_i(X) = \phi_{\pi(i)}(\pi X), \quad i \in N.$$

Finally, we require that in a solution the agents' payoffs are not affected if an independent (payoff-orthogonal) agent is added to the bargaining problem.

Axiom 5 (Separability) *Let* $(N^0, X) \in \mathcal{X}^{SM}$*, where* $N^0 = \{0, 1, \ldots, n\}$*. Denote* $N' = N^0 \cup \{n+1\}$ *and* $X' = X \times [a, b]$*,* $0 \leq a \leq b$*, If* $(N', X') \in \mathcal{X}^{SM}$*, then* $\phi_i(X') = \phi_i(X)$ *for all* $1 \leq i \leq n$*.*

It can be verified that Axioms 1 – 5 are independent.

[6] However, adding an outcome may affect the solution.

6 The Solution

We next characterize the solution on $\mathcal{X}^{SM}$ which satisfies the above five axioms.

Theorem 1 *A solution ϕ on $\mathcal{X}^{SM}$ satisfies Axioms 1 – 5 if and only if there exists α, $0 \leq \alpha \leq 1$, such that for all (N^0, X) in $\mathcal{X}^{SM}$*

$$\phi_i(X) = \phi_i^\alpha(X) = \alpha d_i(X) + (1-\alpha)MC_i(N^0, X) \quad \textit{for all } i \in N, \tag{4}$$

$$\phi_0(X) = \phi_0^\alpha(X) = \max_{x \in X} \sum\nolimits_{i \in N^0} x_i - \sum\nolimits_{i \in N} \phi_i(X). \tag{5}$$

The proof appears in the Appendix.

The solution of every bargaining problem in $\mathcal{X}^{SM}$ awards every agent in N a weighted average of her individually rational level and her marginal contribution to the grand coalition. The IPRO extracts the remaining surplus. The weights, $(\alpha, 1-\alpha)$, are the same across all agents and across all bargaining problems in $\mathcal{X}^{SM}$. Thus, it is sufficient to determine α for one bargaining problem. The same α then applies to all bargaining problems in $\mathcal{X}^{SM}$, with any number of agents. The parameter α measures the bargaining power of the IPRO: The greater is α, the greater is the payoff of the IPRO.

Example. Consider the following one-agent bargaining problem $\hat{X}_2 = \{(0, x) \in \mathbb{R}^2_+ \mid 0 \leq x \leq 1\}$. The IPRO and the agent, each can guarantee 0 on his own, and together they can achieve 1. By Theorem 1,

$$\begin{aligned} \phi_0^\alpha(\hat{X}_2) &= \alpha, \\ \phi_1^\alpha(\hat{X}_2) &= 1-\alpha. \end{aligned}$$

The theorem asserts that the bargaining power of the IPRO is completely determined by this simple bargaining problem. If the solution for this problem is $\alpha = 1$, then the IPRO obtains the entire surplus of every bargaining problem, leaving the agents only with their individually rational levels. On the other hand, if the solution of this problem is $\alpha = 0$, every agent in every bargaining problem (N^0, X) in $\mathcal{X}^{SM}$ obtains his marginal contribution to the grand coalition, while the IPRO collects the smallest payoff in $ST(X)$. In $\hat{X}_2$ the IPRO and the agent may be regarded as symmetric players. Therefore, $\alpha = 1/2$ could be regarded as a proper division of the surplus. In this case, by Theorem 1, $\alpha = 1/2$ for all problems in $\mathcal{X}^{SM}$. The proposition below shows that, for all $X \in \mathcal{X}^{SM}$, $\phi^{1/2}(X)$ is actually the *nucleolus* of V_X.

Let (N^0, V) be a game in coalitional form. Denote by I_V the set of imputations of V,

$$I_V = \left\{ x \in \mathbb{R}^{N^0} \;\middle|\; \begin{array}{l} \sum_{i \in N^0} x_i = V(N^0), \\ x_i \geq V(i), \text{ all } i \in N^0. \end{array} \right\}.$$

The nucleolus of V is defined as follows (Schmeidler, 1969). For every nonempty set $S \subsetneq N^0$ and every $y \in I_V$ denote the excess of coalition S by

$$e_V(S, y) = V(S) - \sum_{j \in S} y_j. \tag{6}$$

Given $y \in I_V$ define the excess vector $\theta(y) \in \mathbb{R}^{2^{N^0}-2}$ whose components are the excesses $e_V(S, y)$, $S \neq N^0$ and $S \neq \varnothing$, arranged in a decreasing order. The *nucleolus* of the game is the set of payoff vectors $\mathcal{N}_V \subset I_V$ which lexicographically minimizes $\theta(y)$ over I_V. The nucleolus is a singleton and it is in the core of V if the core is nonempty (Schmeidler, 1969).

Proposition 4 *The solution $\phi^{1/2}$ on $\mathcal{X}^{SM}$ is the nucleolus of V_X for every (N^0, X) in $\mathcal{X}^{SM}$.*

The proof appears in the Appendix.

Remark 1 Theorem 1 and the other results which apply to $\mathcal{X}^{SM}$ also apply to a wider class $\mathcal{X}^*$ consisting of all bargaining problems (N^0, X) where the marginal contribution of every agent $i \in N$ to a coalition $S^0 \ni i$ is the smallest for the grand coalition. Formally, $\mathcal{X}^*$ is the set of all bargaining problems (N^0, X) such that for all $S \subset N$ and all $i \in S$

$$MC_i(N^0, X) \leq MC_i(S^0, X).$$

An example of a bargaining problem which is in $\mathcal{X}^*$ but not necessarily submodular is one which involves a limited capacity technology. A small coalition of players can increase its output by adding a player (perhaps, with an increasing rate due to economy of scale) more than a large coalition which has already reached the capacity limit.

Remark 2 A possible alternative definition of the individual rationality is as follows. Suppose that if an agent i unilaterally leaves the bargaining table, the IPRO dictates an outcome $x = (x_0, x_1, \ldots, x_n)$ which is efficient for the players in $N^0 \backslash \{i\}$, i.e., $x \in E_{N^0 \backslash \{i\}}(X)$. In this case, agent i receives x_i. Since $E_{N^0 \backslash \{i\}}(X)$ can contain more than one point, i can guarantee only the minimum level of the i-th component in $E_{N^0 \backslash \{i\}}(X)$. We therefore define $d_i(X) = \min\{x_i : x \in E_{N^0 \backslash \{i\}}(X)\}$.

A more conservative definition takes into account the possibility that i may not be the only one to leave the "bargaining table". In this case, she can only justify a claim of her smallest payoff x_i among all outcomes $x \in X$ which are efficient for S^0, where S varies over all subsets of $N \backslash \{i\}$, i.e., $d_i(X) = \min\{x_i : x \in E_{S^0}(X),\ S \subset N \backslash \{i\}\}$.

Theorem 1 and the other results presented above hold with either of these two alternative definitions of the individual rationality.

Remark 3 We would like to comment on the relationship between our result and that of Buch and Tauman (1992) (thereafter, BT). Let $\mathcal{X}^0 \subset \mathcal{X}$ be the class of bargaining problems with three or more players, where the IPRO can achieve only zero by himself. Formally, $(N^0, X) \in \mathcal{X}^0$ if $|N^0| \geq 3$ and $x_0 = 0$ for all $x \in X$. BT provide an axiomatic approach only to problems in $\mathcal{X}^0$ and find a unique solution, ϕ^{BT}. The BT axiomatic approach is different. It omits the stability and STD axioms and instead imposes the axiom of independence of irrelevant alternatives. For every $(N^0, X) \in \mathcal{X}^0$, BT define the individually rational level of every agent $i \in N$ as

$$d_i(X) = \min\{x_i : x \in E_{N^0 \setminus \{i\}}(X)\}. \tag{7}$$

The unique solution of BT is

$$\begin{aligned} \phi_i^{BT}(X) &= d_i(X) \text{ for all } i \in N, \\ \phi_0^{BT}(X) &= V_X(N^0) - \sum\nolimits_{i \in N} d_i(X), \end{aligned}$$

Namely, each agent receives only his individually rational level, and the IPRO (the *ruler*, in BT) obtains the surplus. Note that the solution ϕ^{BT} coincides with our solution[7] ϕ^α for $\alpha = 1$ on $\mathcal{X}^{SM} \cap \mathcal{X}^0$.

7 Two Examples

7.1 A Monopoly Industry with an Entry Barrier

Consider a monopoly industry with a technological entry barrier. Namely, there is a monopolist (player 1) and $n-1$ potential entrants (players $2, 3, \dots, n$), $n \geq 3$. Suppose that the monopolist possesses the exclusive right for some production technology; the potential entrants have access to an inferior technology which does not enable them to compete with the monopolist.

Let player 0, the IPRO, be an outside innovator who possesses a new technology which is as efficient as the monopolist's technology. The IPRO licenses his technology to a subset of firms of his choice. A licensee firm has the same cost function as the monopolist.

[7] Provided $d(\cdot)$ is given by (7) (see also Remark 2 above).

Let $N = \{1, 2, \ldots, n\}$, let $K \subset N \backslash \{1\}$ be the set of licensee firms, and denote $k = |K|$. Let q_i be the quantity produced by firm i and let $Q = \sum_{i \in N} q_i$. The cost function of every licensee i in K is the same as the cost function of the incumbent monopolist and is given by $C(q_i) = cq_i$. The only producers are the firms in $K \cup \{1\}$. The inverse demand function for the product is linear, $P(Q) = \max\{0, a - Q\}$, where $a > c > 0$.

We next describe the bargaining problem (N^0, X) and compute its solution $\phi^\alpha(X)$, $0 \leq \alpha \leq 1$. The set of players is $N^0 = N \cup \{0\}$. The set of outcomes $X \subset \mathbb{R}^{n+1}$ consists of $(n+1)$-tuples of the form $x(k) = (x_0(k), \ldots, x_n(k))$, for any k, $0 \leq k \leq n-1$, where $x_i(k)$ is the Cournot profit of firm i, $i \in K \cup \{1\}$, and $x_j(k) = 0$ for every non-licensee j, $j \neq 1$. It is straightforward to show that for every $i \in K \cup \{1\}$,

$$x_i(k) = \left(\frac{a-c}{k+2}\right)^2.$$

Every firm in a coalition S^0 containing the IPRO has access to the new technology and may become a licensee. Suppose that S^0 contains the incumbent monopolist, i.e., $1 \in S^0$. Then the maximum profit that S^0 can obtain is the monopoly profit, $V_X(S^0) = (a-c)^2/4$, which is achieved for $k = 0$, namely, by giving no licenses.

Next, suppose that $1 \notin S^0$. Then the maximum total payoff of S^0 is given by

$$V_X(S^0) = \max_{1 \leq k \leq |S|} k \cdot \left(\frac{a-c}{k+2}\right)^2.$$

If $|S| \geq 2$, this is maximized for $k = 2$, and thus

$$V_X(S^0) = (a-c)^2/8.$$

We can now compute the marginal contribution $MC_i(N^0, X)$ of every player $i \in N$. For the incumbent monopolist, we have

$$\begin{aligned} MC_1(N^0, X) &= V_X(N^0) - V_X(N^0 \backslash \{1\}) \\ &= (a-c)^2/4 - (a-c)^2/8 = (a-c)^2/8. \end{aligned}$$

For every other firm $i = 2, \ldots, n$, we have $MC_i(N^0, X) = V_X(N^0) - V_X(N^0 \backslash \{i\}) = 0$, since both N^0 and $N^0 \backslash \{i\}$ contain the incumbent monopolist.

The individually rational level $d_i(X)$ of firm $i \in N$ is the profit that i can guarantee no matter who has access to the new technology. For every potential entrant $i = 2, \ldots, n$, being a non-licensee and receiving zero profit is the worst case, thus $d_i(X) = 0$. For the incumbent monopolist, the worst case is when all firms use the new technology, i.e.,

$$d_1(X) = \left(\frac{a-c}{n+1}\right)^2.$$

Since for all $0 \leq \alpha \leq 1$, the solution ϕ^α is efficient, the IPRO dictates the outcome which maximizes the industry profit. Thus the incumbent monopolist will remain the only producer, and the innovation is "shelved". The net payoffs are given by

$$\begin{aligned}
\phi_1^\alpha(X) &= \alpha \left(\frac{a-c}{n+1}\right)^2 + (1-\alpha)\frac{(a-c)^2}{8}, \\
\phi_i^\alpha(X) &= 0, \quad i = 2, \ldots, n, \quad \text{and} \\
\phi_0^\alpha(X) &= V_X(N^0) - \sum\nolimits_{i=1}^n \phi_i^\alpha(X) \\
&= (1+\alpha)\frac{(a-c)^2}{8} - \alpha \left(\frac{a-c}{n+1}\right)^2.
\end{aligned}$$

Notice that when the bargaining power of the IPRO is minimal, $\alpha = 0$, the IPRO obtains a half of the monopoly profit; with the maximal bargaining power, $\alpha = 1$, he obtains

$$\phi_0^1(X) = \frac{(a-c)^2}{4} - \frac{(a-c)^2}{(n+1)^2},$$

which is at least $3/4$ of the monopoly profit (for $n = 3$), approaching the entire monopoly profit as $n \to \infty$. Thus, in every solution ϕ^α, the incumbent monopolist pays to the IPRO at least $1/2$ of his profit to ensure "shelving" of the new technology.

7.2 An Oligopoly Industry with Identical Firms

Consider a Cournot oligopoly industry with $n+1$ identical firms, $N^0 = \{0, 1, \ldots, n\}$, producing a single good with a constant return to scale technology. Let c be the (fixed) marginal cost of production. The inverse demand function for the product is linear, $P(Q) = \max\{0, a - Q\}$, where $a > c > 0$. Player 0 is an incumbent innovator who, besides producing with his superior technology, may also license it to any subset of firms. The new technology reduces the marginal cost of every licensee from c to $c - \varepsilon$, $\varepsilon > 0$. The set of outcomes $X \subset \mathbb{R}^{n+1}$ consists of the vectors $x(k)$, where $x_i(k)$ is the Cournot profit of firm i, $i \in N^0$, and k is the number of licensees (including the incumbent innovator), $1 \leq k \leq n+1$.

Let k^* be the number of licensees maximizing the industry profit, and suppose that $n \geq 2(\frac{a-c}{\varepsilon} - 1)$. It can be verified that[8] $k^* = \frac{a-c}{\varepsilon}$ (see, e.g., Kamien and Tauman, 2002). This is the minimal number of licensees that drives the market price to c, the pre-innovation marginal cost. Hence, every non-licensee firm exits the market. Every producing firm obtains a per-unit profit ε and produces ε units (the total demand is $a - c$, and $(a-c)/k^* = \varepsilon$), thus receiving the profit of ε^2. The total industry profit is $(a-c)\varepsilon$.

Let us now compute the solution $\phi^a(X)$, $0 \leq \alpha \leq 1$. The marginal contribution of every firm, except for the innovator, is zero. The reason is that for every $i \in N$, $N_0 \backslash \{i\}$ includes more than k^* firms, and when k^* of them have access to the new technology, the firms in $N_0 \backslash \{i\}$ receive the total profit of $(a-c)\varepsilon$, while firm i is forced to exit. Hence, for all $i \in N$, $MC_i(X) = 0$ and, clearly, $d_i(X) = 0$. Therefore, for every $\alpha \in [0, 1]$,

$$\phi_i^\alpha(X) = \alpha d_i(X) + (1-\alpha) MC_i(X) = 0, \quad i \in N,$$
$$\phi_0^\alpha(X) = (a-c)\varepsilon.$$

[8] For simplicity we assume that $\frac{a-c}{\varepsilon}$ is an integer.

It turns out that this result coincides with the non-cooperative result of Kamien and Tauman (2002), where the innovator sells licenses by an auction. The innovator chooses a number k and auctions off k licenses. The k highest bidders win and use the new technology. The innovator collects their bids. If $n \geq 2(\frac{a-c}{\varepsilon} - 1)$, it is optimal to auction off $\frac{a-c}{\varepsilon}$ licenses, and the innovator again extracts $(a-c)\varepsilon$.

Our result is also consistent with Tauman and Watanabe (2007), who obtained the same equivalence result for the Shapley value, this time in the limit when n increases indefinitely.

8 Conclusion

In this paper we provide solutions to bargaining problems involving an IPRO and a set of players. We impose five axioms and characterize the solutions on the class of all submodular bargaining problems. Any solution assigns every agent an average of her individually rational level and her marginal contribution to all other players. The weights defining this average are the same across all agents and across all submodular problems. Thus, they can be used to measure the bargaining power of the IPRO. The higher is the weight assigned to the individually rational level of an agent, the higher is the bargaining power of the IPRO. When he has the full bargaining power, every agent obtains her individually rational level only, and the IPRO, who dictates an efficient outcome, obtains the rest of the “cake”. If the IPRO has the weakest bargaining power, every agent obtains her marginal contribution. If the IPRO and every agent have equal bargaining power, the solution coincides with the nucleolus of a naturally related coalitional game.

A possible direction which we find interesting to explore is bargaining with several IPRO-like entities (bureaucrats). The bureaucrats can dictate any outcome (for instance, by unanimity or by majority vote). Even the case of a single agent and multiple bureaucrats seems to be nontrivial. Another interesting direction is to analyze bargaining problems where the IPRO can dictate a *subset* of outcomes (not a specific outcome) which is an element of a given partition of the set of all outcomes.

Appendix

Proof of Proposition 2

Let $(N^0, X) \in \mathcal{X}$ and let (N^0, V_X) be the associated game in coalitional form. By construction of V_X we obtain that $y \in \mathcal{C}_{V_X}$ if and only if it satisfies
(i) $\sum_{i\in N^0} y_i = \max_{x\in X} \sum_{i\in N^0} x_i$,
(ii) $\sum_{i\in S^0} y_i \geq \max_{x\in X} \sum_{i\in S^0} x_i$ for all $S^0 \subset N^0$, and
(iii) $y_i \geq d_i(X)$ for all $i \in N^0$.

We shall show that $y \in ST(X)$ if and only if it satisfies (i) – (iii). Note that (i) is implied by (ii) for every $y \in Y(X)$ (see Corollary 1). If $y \in ST(X)$, then (i) and (ii) are satisfied by Proposition 1 and (iii) is satisfied because $y \in Y(X)$. Conversely, if y satisfies (i) – (iii) and $y \in Y(X)$, then $y \in ST(X)$ by Proposition 1. The only part which is left to prove is that if $y \in \mathbb{R}^{N^0}$ satisfies (i) – (iii), then $y \in Y(X)$. Let $x^* \in E(X)$ and $x^{N^0\backslash i} \in E_{N^0\backslash i}(X)$. By (i) and (ii), for all $i \in N$,

$$\begin{aligned} y_i &= \sum_{j\in N^0} x_j^* - \sum_{j\in N^0\backslash i} y_j \leq \sum_{j\in N^0} x_j^* - \sum_{j\in N^0\backslash i} x_j^{N^0\backslash i} \\ &= x_i^* + \sum_{j\in N^0\backslash i} x_j^* - \sum_{j\in N^0\backslash i} x_j^{N^0\backslash i} \leq x_i^*. \end{aligned}$$

By (iii), $y_i \geq d_i(X)$. Hence, $y \in Y(x^*) \subset Y(X)$.

Lemmata

We make use of the following two lemmata. The proofs are straightforward, and thus omitted. The number of elements of $S \subset N$ will be denoted by s.

Lemma 2 *Let* (N^0, X) *and* (N^0, X') *be in* $\mathcal{X}$*. Suppose that for some* $b = (b_0, b_1, \ldots, b_n) \in \mathbb{R}^{N^0}$ *and* $c \in \mathbb{R}_{++}$*,* $X' = cX + b$*. Then*

$$\begin{aligned} V_{X'}(S^0) &= cV_X(S^0) + \sum\nolimits_{j \in S^0} b_j, \quad S^0 \subset N^0, \quad \textit{and} \\ d_i(X') &= cd_i(X) + b, \quad i \in N^0. \end{aligned}$$

Lemma 3 *Let* $(N^0, X) \in \mathcal{X}$*, where* $N^0 = \{0, 1, \ldots, n\}$*. Let* $N' = N^0 \cup \{n+1\}$ *and* $X' = X \times [a, a']$*, where* $0 \leq a \leq a'$*. Then*

$$\begin{aligned} V_{X'}(S^0) &= V_X(S^0), \quad S^0 \subset N^0, \quad \textit{and} \\ d_i(X') &= d_i(X), \quad i \in N. \end{aligned}$$

Proof of Theorem 1

Existence. By Lemma 1, ϕ satisfies Stability and STD axioms. To verify the Scale Covariance axiom, let (N^0, X) and (N^0, X') be in $\mathcal{X}^{SM}$ such that for some $\hat{b} \in \mathbb{R}^{N^0}$ and $\hat{c} \in \mathbb{R}_{++}$, $X' = \hat{c}X + \hat{b}$. By Lemma 2, for all $i \in N$, $d_i(X') = \hat{c}d_i(X) + \hat{b}_i$, $MC_i(X') = \hat{c}MC_i(X) + \hat{b}_i$, and $V_{X'}(N^0) = \hat{c}V_X(N^0) + \sum_{j \in N^0} \hat{b}_j$. Therefore, for all $i \in N$, $\phi_i(X') = \hat{c}\phi_i(X) + \hat{b}_i$, and

$$\begin{aligned} \phi_0(X') &= \hat{c}V_X(N^0) + \sum\nolimits_{j \in N^0} \hat{b}_j - \sum\nolimits_{j \in N^0} (\hat{c}\phi_j(X) + \hat{b}_j) \\ &= \hat{c}\left(V_X(N^0) - \sum\nolimits_{j \in N^0} \phi_j(X)\right) + \hat{b}_0 = \hat{c}\phi_0(X) + \hat{b}_0. \end{aligned}$$

The Anonymity axiom is trivially satisfied. Finally, we verify the separability axiom. Let $(N^0, X) \in \mathcal{X}^{SM}$, where $N^0 = \{0, 1, \ldots, n\}$. Let $N' = N^0 \cup \{n+1\}$ and $X' = X \times [a, a']$, where $0 \leq a \leq a'$. Clearly, $(N', X') \in \mathcal{X}^{SM}$. By

Lemma 3, for all $i \in N$, $d_i(X') = d_i(X)$, $MC_i(N^0, X') = MC_i(N', X)$, and $V_{X'}(N^0) = V_X(N^0)$, implying that $\phi(X') = \phi(X)$.

Uniqueness (up to the parameter α**).** Let ϕ be a solution on $\mathcal{X}^{SM}$ which satisfies Axioms 1 – 5. Let

$$\hat{X}_2 = \{(0, x) \mid 0 \leq x \leq 1\}$$

and let $\phi_0(\hat{X}_2) = \alpha$. Since $ST(\hat{X}_2) = \{y \in \mathbb{R}^2_+ \mid y_0 + y_1 = 1\}$, it must be that $\phi_1(\hat{X}_2) = 1 - \alpha$. We shall show that $\phi(X)$ is uniquely determined, given α, for all $X \in \mathcal{X}^{SM}$.

Consider next the bargaining problem in $\mathcal{X}^{SM}_2$ defined by

$$X_{(d,b)} = \{d_0\} \times [d_1, b_1],$$

where $d = (d_0, d_1) \in \mathbb{R}^2_+$ and $b_1 \geq d_1$. Clearly, $X_{(d,b)} = d + (b_1 - d_1)\hat{X}$, and, by the Scale Covariance axiom,

$$\phi(X_{(d,b)}) = d + (b_1 - d_1)(\alpha, 1 - \alpha),$$

and $\phi(X_{(d,b)})$ is uniquely determined. Next, consider the bargaining problem $(N^0, \bar{X}_{(d,b)}) \in \mathcal{X}^{SM}$, where $d = (d_0, d_{1,} \ldots, d_n) \in \mathbb{R}^{N^0}_+$ and $b = (b_1, \ldots, b_n) \in \mathbb{R}^N_+$ such that $b_i \geq d_i$ for all $i \in N$, and

$$\bar{X}_{(d,b)} = \{d_0\} \times [d_1, b_1] \times \ldots \times [d_n, b_n].$$

By the Separability and Anonymity axioms, for every $i \in N$,

$$\phi_i(\bar{X}_{(d,b)}) = \alpha d_i + (1 - \alpha)b_i.$$

This, together with the fact that $\phi(\bar{X}_{(d,b)})$ is efficient, uniquely determines $\phi(\bar{X}_{(d,b)})$. Also observe that

$$ST(\bar{X}_{(d,b)}) = \left\{ y \in R^{N^0}_+ \;\middle|\; \begin{array}{l} d_i \leq y_i \leq b_i \text{ for all } i \in N, \\ y_0 = d_0 + \sum_{i \in N}(b_i - y_i) \end{array} \right\}.$$

Let (N^0, X) be an arbitrary bargaining problem in $\mathcal{X}^{SM}$. Let $\hat{d}_i = d_i(X)$ and $\hat{b}_i = MC_i(N^0, X)$, $i \in N$. Also, let $\hat{d}_0 = V_X(N^0) - \sum_{i\in N} \hat{b}_i$. Then, by Lemma 1,

$$ST(X) = \left\{ y \in R_+^{N^0} \;\middle|\; \begin{array}{l} \hat{d}_i \leq y_i \leq \hat{b}_i \text{ for all } i \in N, \\ y_0 = \hat{d}_0 + \sum_{i\in N}(\hat{b}_i - y_i) \end{array} \right\} = ST(\bar{X}_{(\hat{d},\hat{b})}).$$

Since $ST(X) = ST(\bar{X}_{(\hat{d},\hat{b})})$, by the STD axiom, $\phi(X) = \phi(\bar{X}_{(\hat{d},\hat{b})})$, and $\phi(X)$ is uniquely determined for every $X \in \mathcal{X}^{SM}$. This completes the proof.

Proof of Proposition 4

Let $(N^0, X) \in \mathcal{X}^{SM}$. Then for every $S \subset N$ and every $i \in N\backslash S$, $V_X(N^0) - V_X(N^0\backslash i) \leq V_X(N^0\backslash S) - V_X(N^0\backslash(S \cup i))$, or

$$\sum_{j\in S} V_X(N^0) - V_X(N^0\backslash j) \leq V_X(N^0) - V_X(N^0\backslash S). \tag{8}$$

For every $y \in Y(X)$ and every $S \subset N^0$ define

$$e_X(y, S) = V_X(S) - \sum_{i\in S} y_i.$$

First, note that for every $S \subset N$, $V_X(S) = \sum_{i\in S} d_i(X)$, hence, for all $y \in Y(X)$,

$$e_X(y, S) = \sum_{i\in S} e_X(y, \{i\}). \tag{9}$$

Next, for every $S \subset N$ and every $y \in Y(X)$, by (8),

$$\begin{aligned} e_X(y, N^0\backslash S) &= V_X(N^0\backslash S) - \sum_{i\in N^0\backslash S} y_i = V_X(N^0\backslash S) - V_X(N^0) + \sum_{i\in S} y_i \\ &\leq \sum_{i\in S} \big(V_X(N^0\backslash i) - V_X(N^0) + y_i\big) \\ &= \sum_{i\in S} \left(V_X(N^0\backslash i) - \sum_{j\in N^0\backslash i} y_j\right) = \sum_{i\in S} e_X(y, N^0\backslash i). \end{aligned} \tag{10}$$

By (9) and (10), for all $y \in Y(X)$ and all $S \subset N$,

$$\begin{aligned}\sum_{i \in S} e_X(y, \{i\}) &\geq e_X(y, S), \\ \sum_{i \in S} e_X(y, N^0 \backslash i) &\geq e_X(y, N^0 \backslash S).\end{aligned}$$

Therefore, the nucleolus y^* of V_X is defined for every $i \in N$ by

$$y_i^* = \underset{y \in ST(X)}{\operatorname{argmin}} \left[\max\left\{e_X(y, \{i\}), e_X(y, N^0 \backslash i)\right\}\right].$$

Since $e_X(y, \{i\}) = d_i(X) - y_i$ and $e_X(y, N^0 \backslash i) = V_X(N^0 \backslash i) - V_X(N^0) + y_i$, y_i^* is the solution of

$$d_i(X) - y_i = V_X(N^0 \backslash i) - V_X(N^0) + y_i.$$

Thus,

$$y_i^* = \frac{V_X(N^0) - V_X(N^0 \backslash i) + d_i(X)}{2} = \frac{MC_i(N^0, X) + d_i(X)}{2}, \quad i \in N.$$

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References

Bernheim, B. D. and M. D. Whinston (1986). Menu auctions, resource allocation, and economic influence. *Quarterly Journal of Economics 101*, 1–32.

Buch, I. and Y. Tauman (1992). Bargaining with a ruler. *International Journal of Game Theory 21*, 131–148.

Kamien, M. (1992). Patent licensing. In R. J. Aumann and S. Hart (Eds.), *Handbook of Game Theory*, Volume 1, pp. 332–354. North-Holland.

Kamien, M. and Y. Tauman (2002). Patent licensing: the inside story. *The Manchester School 70*, 7–15.

Kamien, M., Y. Tauman, and S. Zamir (1990). On the value of information in a strategic conflict. *Games and Economic Behavior 2*, 129–153.

Nash, J. (1950). The bargaining problem. *Econometrica 18*, 155–162.

Schmeidler, D. (1969). The nucleolus of a characteristic function game. *SIAM Journal of Applied Mathematics 17*, 1163–1170.

Sen, D. and Y. Tauman (2007). General licensing schemes for a cost-reducing innovation. *Games and Economic Behavior 59*, 163–186.

Tauman, Y. and N. Watanabe (2007). The Shapley value of a patent licensing game: the asymptotic equivalence to non-cooperative results. *Economic Theory 30*, 135–149.